

A model of non-perturbative gluon emission in an initial state parton shower

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ABSTRACT: We consider a model of transverse momentum production in which non-perturbative smearing takes place throughout the perturbative evolution, by a simple modification to an initial state parton shower algorithm. Using this as the important non-perturbative ingredient, we get a good fit to data over a wide range of energy. Combining it with the non-perturbative masses and cutoffs that are a feature of conventional parton showers also leads to a reasonable fit. We discuss the extrapolation to the LHC.

KEYWORDS: Quantum Chromodynamics, Non-perturbative physics, Drell-Yan process.

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1. Introduction

At LEP and the SLC, the properties of W and Z bosons could be studied to great accuracy because to a very good approximation they could be calculated using only electroweak perturbation theory. At the current Tevatron and future LHC colliders, on the other hand, the event rates are enormous and the expected statistical precision excellent, but the electroweak bosons are produced by the annihilation of coloured partons that are initially confined into colourless hadrons. This means that QCD effects, both perturbative and non-perturbative, play an extremely important role in determining the properties of events containing electroweak bosons and the limited precision with which we can calculate those effects will ultimately be responsible for the dominant systematic uncertainties on the measurements.

In this paper we will concentrate on one particular property of the produced W and Z bosons¹, namely their transverse momentum distribution. This is interesting from the QCD point of view as, sweeping across the distribution, one has regions

¹We are also interested in virtual photons with invariant masses well below that of the Z, particularly for tuning and validating our model. All our calculations include properly the full interference between γ^* and Z, but with an eye on the ultimate application at the LHC, we continue to refer in this introduction to Zs.

dominated by hard perturbative emission, multiple soft and/or collinear, but still perturbative, emission, and truly non-perturbative confinement effects. It is also an important quantity for the experimental programme, because the W reconstruction efficiency is transverse momentum dependent, having a direct effect on the ultimate precision of the W mass measurement as well as helping the understanding of the signature for Higgs boson production at either the Tevatron or the LHC [1]. Although the experiments measure the Z transverse momentum distribution and use this to infer that of the W, the extent to which the effects are non-universal limits the ultimate accuracy of the measurement, unless elaborate tricks as proposed in Ref. [2] are used. Thus, a deeper theoretical understanding and more reliable models are certainly needed.

The two approaches to predicting the transverse momentum distribution are analytical resummation [3–9] and parton shower algorithms [10–12] (there have also been attempts to combine the two approaches [13]). We will focus on the latter, but will draw a few comparisons with the former later. The parton shower approach starts from the tree-level matrix element, usually supplemented by ‘matrix element corrections’ [12, 14–18] that use higher-order tree-level matrix elements to describe emission at scales of order the W/Z boson mass and higher². These give a significant tail of events with very high transverse momenta. The hard events are then evolved down to low scales by using the backward evolution parton shower approach [10]. Recoil from the gluons emitted³ during this evolution build up a transverse momentum for the W/Z. The evolution terminates at some scale of order the confinement or typical hadron mass scale. However, confinement effects, described for example as the Fermi motion of partons within the hadron, mean that the partons initiating the shower should have a non-perturbative transverse momentum distribution, often described as their ‘intrinsic’ transverse momentum, which is also transferred to the W/Z by recoil [4].

One way to implement the colour coherence inherent in QCD is to formulate the parton shower as an evolution in (energy times) opening angle, as implemented in the HERWIG [20] and Herwig++ [21–23] event generators. Analysis of higher order corrections shows that the scale of the running coupling used in this evolution should be of order the transverse momentum of the emission [24, 25], and once this is done one must introduce an infrared cutoff in transverse momentum that is active during every step of the evolution. That is, the probability of each backward step in the evolution variable, even at large values of that variable, is logarithmically dependent on the cutoff. In Ref. [26], one of us advocated the view that conventional infrared cutoff scales on perturbative emission (in that case on the transverse momenta used to describe the minijet production in an underlying event model) should be thought of as

²We do not go into their details, but use the implementation of [19] throughout this paper.

³Together with other backward-evolution steps such as an incoming sea quark being evolved back to an incoming gluon by emitting a corresponding antiquark.

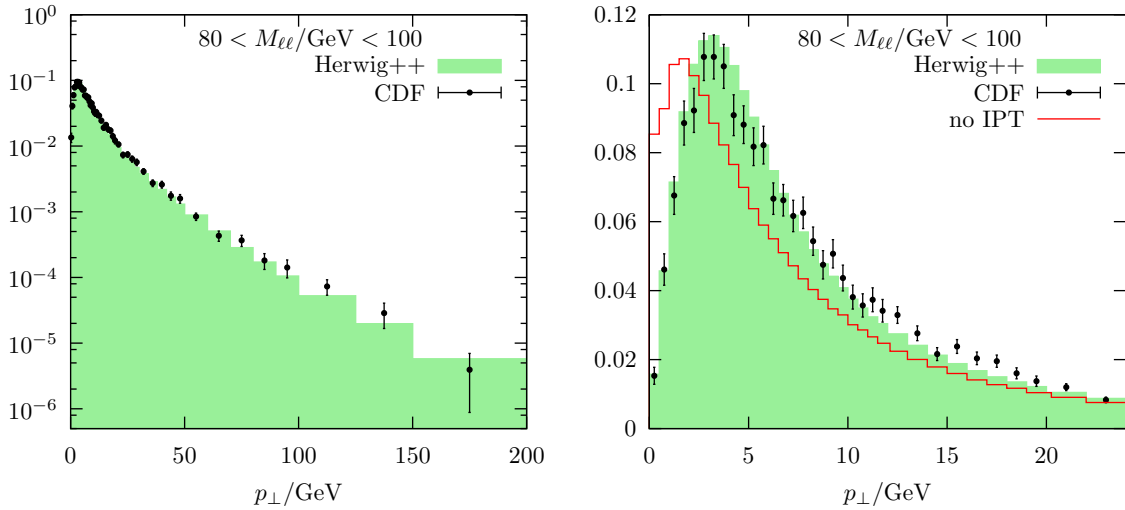


Figure 1: The transverse momentum distribution of Z bosons at Tevatron energies compared to CDF data. Up to large transverse momenta (left) and only the small p_{\perp} region (right). The line denoted “no IPT” is from Herwig++ with intrinsic transverse momentum off.

infrared *matching* scales, with a non-perturbative model of emission below the cutoff supplementing the usual perturbative one above. In this paper we propose such a model for backward evolution in which an additional non-perturbative component at low transverse momentum provides additional smearing at each step of the evolution.

We are particularly motivated by the fact that, in order to fit data, conventional parton shower models need an ‘intrinsic’ transverse momentum $\langle k_T \rangle$ that grows with collision energy. For example in Herwig++ its value grows from $\langle k_T \rangle = 0.9$ GeV which is needed to describe the data taken at the energy $\sqrt{S} = 62$ GeV (experiment R209) to 2.1 GeV which is needed at the Tevatron energies ($\sqrt{S} = 1800$ GeV). One would expect the average ‘intrinsic’ transverse momentum per parton to be of the order of $0.3 - 0.5$ GeV based solely on the proton size and uncertainty rule, but the values extracted from data, even with attempts to reduce its value [27] are too large and cannot be interpreted as “intrinsic”. As we shall see, different models of this energy dependence that fit current data give very different predictions for the LHC. In our model, this growth is under some kind of ‘semi-perturbative’ control, since the amount of non-perturbative smearing grows with the length of the perturbative evolution ladder. We ask the question whether, with this additional source of non-perturbative transverse momentum, a truly intrinsic transverse momentum distribution for the initial partons, that does not depend on the collision energy or type, is sufficient.

In Fig. 1 we show a comparison of the Z-boson transverse momentum spectrum at Tevatron Run I with CDF data [28]. The left panel shows that a description is possible up to large transverse momentum. The high transverse momentum region

is, however, dominated by contributions from hard gluon emissions. These will not be the focus of this paper. In general, the large transverse momentum region will not be affected by soft, non-perturbative emissions.

In the right panel of Fig. 1 we see only the small transverse momentum region. The Herwig++ result is shown with an intrinsic $p_\perp = 2.1$ GeV from Gaussian smearing [19], which is the default value at Tevatron energies. To show the importance of this effect we also plot the result with intrinsic p_\perp set to zero. Clearly, this non-perturbative Gaussian smearing only affects the region of small transverse momenta. At large boson p_\perp the recoil against hard, perturbative gluon radiation dominates the spectrum.

We also compared to D0 data [29] and found a similar agreement. However, the CDF data has a finer binning and is therefore more suitable for our comparison.

2. Description of the model

In order to simulate non-perturbative emission with the parton shower, we consider the Sudakov form factor for backward evolution from some scale \tilde{q}_{\max} down to \tilde{q} that is implemented in the parton shower Monte Carlo program Herwig++. For further details, cf. Ref. [30]

$$\Delta(\tilde{q}; p_{\perp\max}, p_{\perp 0}) = \exp \left\{ - \int_{\tilde{q}^2}^{\tilde{q}_{\max}^2} \frac{d\tilde{q}'^2}{\tilde{q}'^2} \int_{z_0}^{z_1} dz \frac{\alpha_S(p_\perp)}{2\pi} \frac{x' f_b(x', \tilde{q}'^2)}{x f_a(x, \tilde{q}'^2)} P_{ba}(z, \tilde{q}'^2) \right\}, \quad (2.1)$$

with $x' = x/z$. The argument of the strong coupling α_S in Eq. (2.1) is the transverse momentum p_\perp of an emission⁴. The cut-off scale at which the coupling would diverge, if extrapolated outside the perturbative domain is represented by $p_{\perp 0}$. Therefore two arguments of the Sudakov formfactor, $p_{\perp\max}$ and $p_{\perp 0}$ are not the evolution variables but only explicitly denote the available phase-space of an emission.

We can introduce additional non-perturbative emissions in terms of an additional Sudakov form factor Δ_{NP} , such that we have

$$\Delta(\tilde{q}; p_{\perp\max}, 0) = \Delta_{\text{pert}}(\tilde{q}; p_{\perp\max}, p_{\perp 0}) \Delta_{NP}(\tilde{q}; p_{\perp 0}, 0) \quad (2.2)$$

For technical simplicity we can achieve this by modifying our implementation of $\alpha_S(p_\perp)$ in such a way that we can extend it into the non-perturbative region,

$$\alpha_S(p_\perp) = \alpha_S^{(\text{pert})}(p_\perp) + \alpha_S^{(\text{NP})}(p_\perp). \quad (2.3)$$

⁴Generally the scale of α_S is a function of the evolution variables z and \tilde{q}^2 and by default in Herwig++, the argument of α_S is a slightly simplified expression, equal to the transverse momentum to the required accuracy, but not exactly. We have tested the implementation of our model with this simplified expression and the exact expression for transverse momentum, and find very similar results. We therefore use the default expression.

In our implementation we have not chosen $\alpha_S^{(\text{pert})}(p_\perp)$ and $\alpha_S^{(\text{NP})}(p_\perp)$ explicitly but rather modified the sum $\alpha_S(p_\perp)$, in order to behave differently in two physically different regions, divided by a separation scale p_{\perp_0} ,

$$\alpha_S(p_\perp) = \begin{cases} \varphi(p_\perp), & p_\perp < p_{\perp_0} \\ \alpha_S^{(\text{pert})}(p_\perp), & p_\perp \geq p_{\perp_0} \end{cases}. \quad (2.4)$$

In this way, the kinematics and phase space of each non-perturbative emission are exactly as in the perturbative case. We only modify their probabilities in the region of small transverse momenta.

We have studied two simple choices of the non-perturbative function $\varphi(p_\perp)$ in greater detail:

- (a) “*flat*”: the flat continuation of $\alpha_S(p_\perp < p_{\perp_0})$ with a constant value $\varphi_0 = \varphi(0)$,

$$\alpha_S(p_\perp < p_{\perp_0}) = \varphi_0. \quad (2.5)$$

- (b) “*quadratic*”: a quadratic interpolation between the two values $\alpha_S(p_{\perp_0})$ and $\varphi(0)$.

$$\alpha_S(p_\perp < p_{\perp_0}) = \varphi_0 + (\alpha_S(p_{\perp_0}) - \varphi_0) \frac{p_\perp^2}{p_{\perp_0}^2}. \quad (2.6)$$

In both cases our model is determined by the two free parameters p_{\perp_0} and φ_0 .

We have concentrated our study on the small transverse momentum region of vector boson production. Therefore, the only modification of the Herwig++ code that had to be made was the introduction of the two non-perturbative parameters to $\alpha_S(p_\perp)$. In fact, as we implemented it this would also affect final state radiation but our observable is not sensitive to effects in the final state. Details of final state effects will be discussed in Sec. 5.2.

We would like to emphasise that we want to keep this model as simple as possible in order to explore the possibility of a reasonable description of the data. Therefore, the shape of α_S in the non-perturbative region is only a crude guess. A further study of the details of the shape would go beyond the scope of this work.

3. Parton-level results

To simulate fully exclusive events, Monte Carlo event generators like Herwig++ use a hadronization model, which is assumed to be universal across different types of collision and different processes within them. Therefore for our final results presented in sections 4 and 5 we will combine our model for non-perturbative gluon emission with the standard Herwig++ model for the termination of the shower using non-perturbative effective parton masses tuned to e^+e^- data so that the corresponding

hadronization model can be used. However, if we are only interested in the W/Z transverse momentum distribution, we do not need to hadronize the final state: we can terminate the simulation at the end of the parton shower. We can therefore make a purely parton-level study with all light quark and gluon effective masses and cutoffs set to zero⁵ with our model for the low-scale α_S as the only non-perturbative input.

The first observation that we can make with our model is that we can easily find parameter values that describe existing Tevatron data. However the main focus of our work is on the understanding of the dependence of the non-perturbative effects on the typical centre of mass (CM) energy of the system or even the collider. We therefore considered two more datasets. The first is Fermilab E605 [31] fixed target p -Cu data, taken at 38.8 GeV CM energy. We only take the data with an invariant mass of $11.5 < M/\text{GeV} < 13.5$ as this goes out to the highest transverse momentum. The other data we consider were taken in p - p collisions at $\sqrt{S} = 62 \text{ GeV}$ at the CERN ISR experiment R209 [32]. There are more data available but all at even lower CM energies. Our main interest is in finding a reasonable extrapolation to LHC energies that is still compatible with the early data.

We have run Herwig++ with varying non-perturbative parameters φ_0 and $p_{\perp 0}$ for the two forms of α_S in (2.5) and (2.6). After an initial broader scan, we focussed on the region of φ_0 between 0 and 1 and $p_{\perp 0}$ between 0.5 GeV and 1.0 GeV. Each parameter set was run for the three different experimental setups we consider. We left the intrinsic k_{\perp} fixed at 0.4 GeV. For each resulting histogram we have computed a total χ^2/bin in order to quantify its agreement or disagreement with the data. We took the data errors to be at least 5% as we did not want to bias towards exceptionally good data points. Furthermore, we ignored an additional systematic error of the two fixed target data sets which is quoted to be around 5–10%. Fig. 2 shows the χ^2 values we obtain for the quadratic model compared to Tevatron data. We made similar plots for the other two energies. The basic features are the same. In each case we find clear minima within the given $p_{\perp 0}$ range. In going from one experiment to another we find the more or less sharp minima. The minimum in Fig. 2 is not as clear as in the cases of the other two experiments. The best and most stable situation for all experiments is found for $\alpha_S(0) = 0.0$ and $p_{\perp 0} = 0.75 \text{ GeV}$. The χ^2 values are not very sensitive to the value $\alpha_S(0)$ around the minimum, i.e. we are not very sensitive to the non-perturbative region itself. In Fig. 4 we show the non-perturbative region of our α_S parametrisation. We have inspected all distributions directly as well and found a

⁵For technical reasons it is not possible to set them exactly to zero. However, we have confirmed that if they are small enough their precise values become irrelevant. For this study we actually set the quark masses and the δ parameter to 1 MeV, so that the non-perturbative mass that cuts off the parton shower, called Q_g in the Herwig++ manual, is given by the cParameter. For the cParameter we ran with values in the range 10 MeV to 100 MeV and found very little effect. We therefore use 100 MeV for our main results.

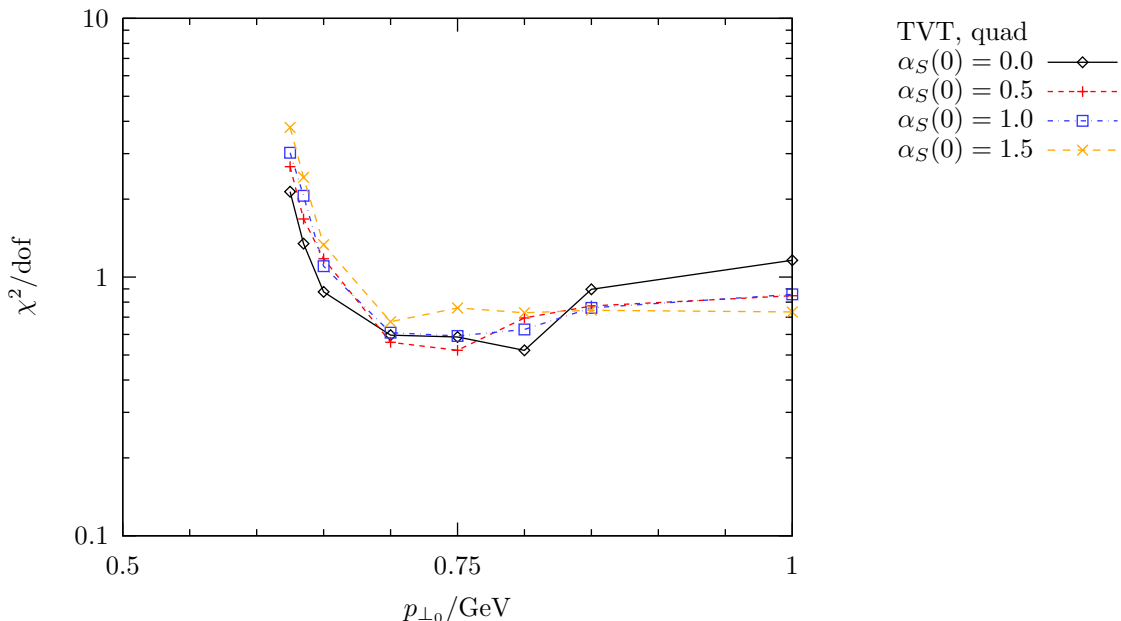


Figure 2: χ^2 values for the quadratic non-perturbative model compared to Tevatron data as a function of the NP parameter $p_{\perp 0}$. The different lines are for different values of $\varphi_0 = \alpha_S(0)$.

consistency with this choice. For this optimal choice over the energy range 38.8 GeV to 1.8 TeV we show the resulting low p_{\perp} distributions in Fig. 3. We should stress that the used parameter set may not be the optimal choice for each experiment or CM energy but rather the best compromise between the three experiments. As the fixed target data do not even include the systematic errors quoted we have deliberately put a bit more emphasis on the Tevatron result. Ultimately, our goal will be to extrapolate our results further to LHC energies and we believe that for this purpose we have made the right choice of parameters.

It is interesting to compare the α_s parametrisation we have found with other approaches to modelling non-perturbative corrections to inclusive observables with a modified coupling in the soft region (see for example Refs. [33–35]). Ref. [33] finds an average value of the coupling over the range from 0 to 2 GeV of about 0.5, while Ref. [34] argues that the effective coupling should vanish at $p_{\perp} \rightarrow 0$. For our best-fit parametrisation, the average value of the coupling over the range from 0 to 2 GeV is around 0.7. Considering that their fits to data typically use NLO calculations, while we have used a leading log parton shower, this could be considered good agreement.

4. Hadron-level results

As we mentioned earlier, the results of the previous section are not suitable for full event simulation, because the masslessness of the light quarks and gluons is

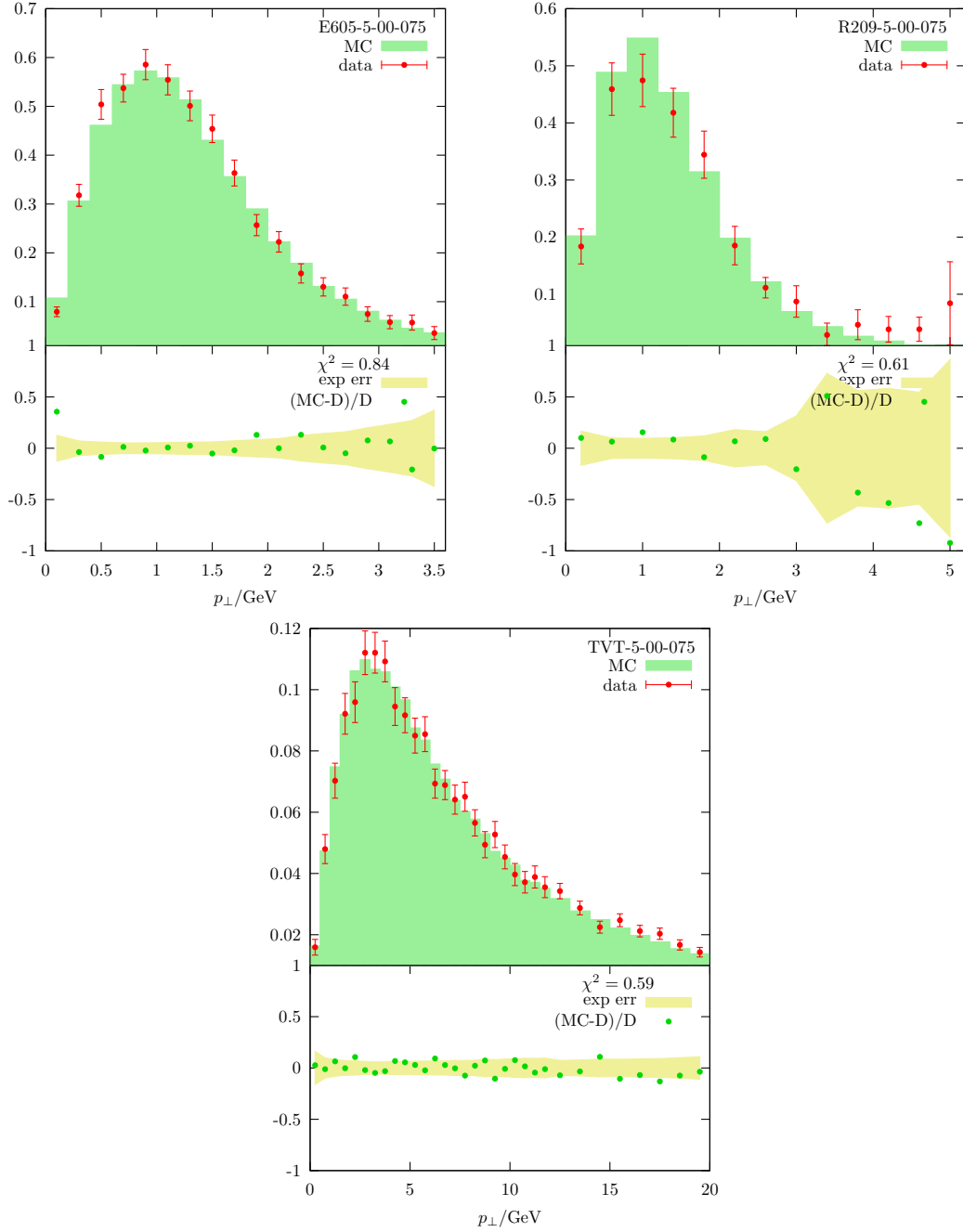


Figure 3: Comparison of the parton level results from the non-perturbative model with data from E605 (top left), R209 (top right) and CDF (bottom). The Monte Carlo results are from our parameter set with $\varphi_0 = 0.0, p_{\perp 0} = 0.75 \text{ GeV}$. Each panel includes two plots. The upper plot compares MC to data directly, whereas the lower plot shows the ratio $(MC-D)/D$ against the relative data error.

not consistent with the hadronization model used in Herwig++. Therefore in this section we perform the same comparison with data but with the effective parton masses returned to their default values, tuned to e^+e^- annihilation data.

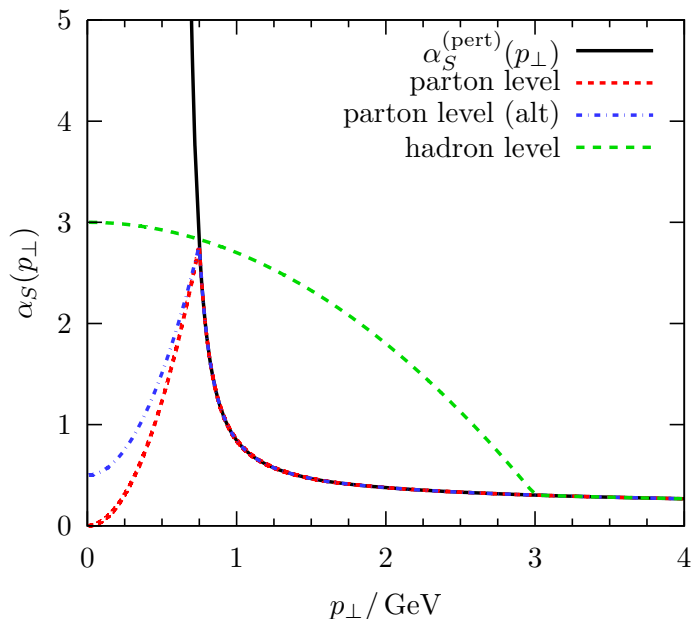


Figure 4: The optimal choice: “quadratic” interpolation with $\alpha_S(0) = 0$ and $p_{\perp 0} = 0.75$ GeV is shown. For comparison, we also show the purely perturbative α_S (LO) and another reasonable parametrisation of α_S in the non-perturbative region for our parton level results. In addition we show our best fit for the hadron level results.

Performing an initial scan over parameter space we find that we need to consider a much wider range of values than in the massless case. We can get a good description of the data from each experiment, but there is more tension between the three experiments leading to a larger total χ^2 . We choose as our best fit point $\alpha_S(0) = 3$ and $p_{\perp 0} = 3.0$ GeV giving a χ^2 per degree of freedom of 0.94 for the Tevatron data and of 2.72 for all data. We show the results in Fig. 5.

This time our best-fit α_S parametrization is very different from those of Refs. [33–35] – it is much larger in the non-perturbative region. This is not surprising since our coupling is now ‘fighting against’ an emission distribution that is already falling as $p_{\perp} \rightarrow 0$ relative to the perturbative one. Although the overall description of data is somewhat worse with the non-perturbative parton masses, it is acceptable, and we prefer to maintain Herwig++’s description of final states so we keep this as our default model for the remainder of the study.

5. LHC result and comparison with other approaches

5.1 Z boson transverse momentum

In this section we would like to compare the result of extrapolating our model to LHC energies with the results from two other approaches: ResBos [36] and Gaussian intrinsic k_{\perp} .

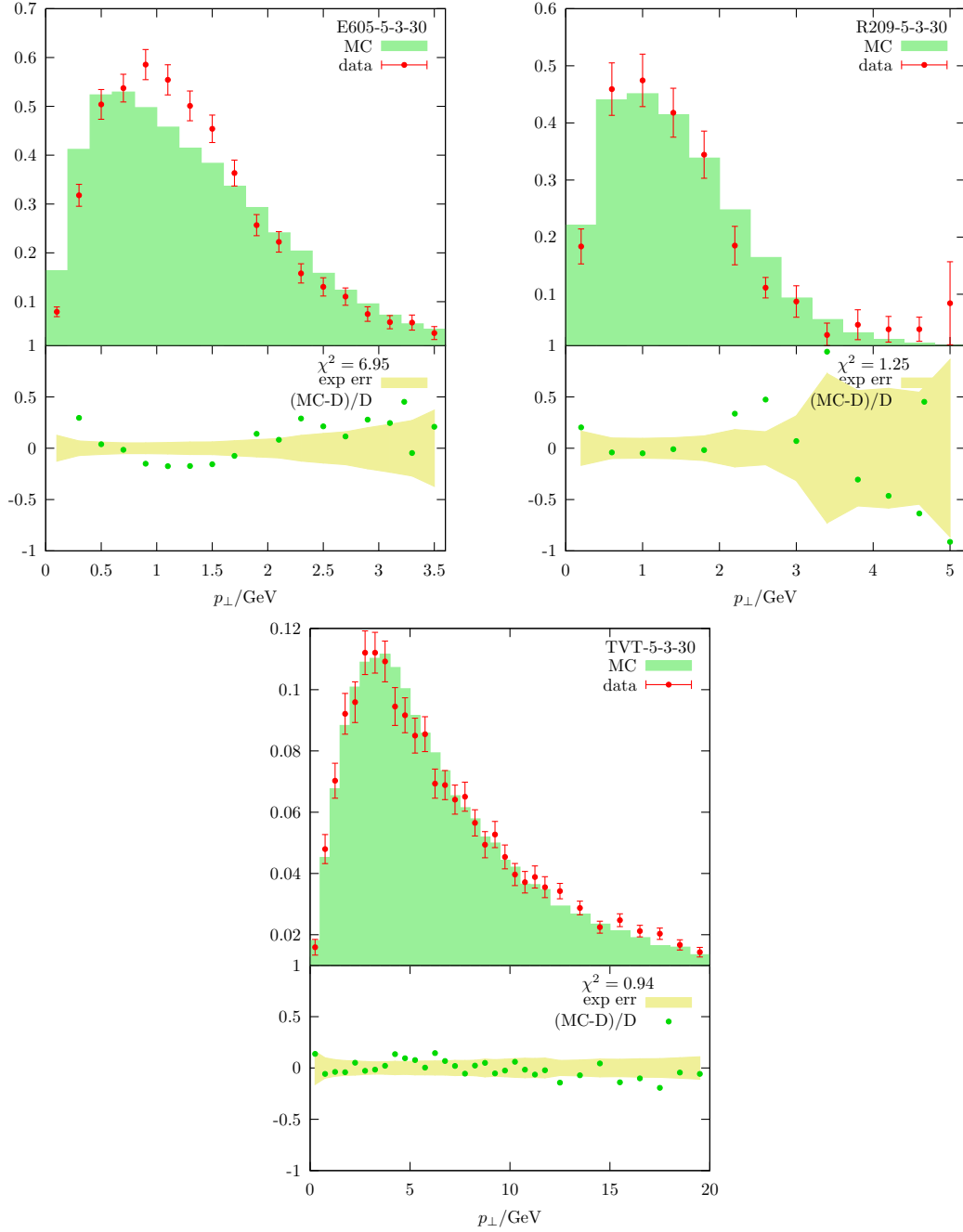


Figure 5: As Fig. 3 but for the combination of our non-perturbative emission model with the model of non-perturbative parton masses built in to Herwig++ by default.

First of all, we compare our prediction on the parton level (filled histogram) and the hadron level (dot-dashed, blue). Both histograms give a consistent extrapolation. We have tried different values of $\alpha_S(0)$, ranging up to 1.5, for our parton level prediction and find no visible effect. This emphasises the relative unimportance of the non-perturbative region for the description of this observable at the LHC.

The result from ResBos in Fig. 6 (solid, black) shows a slightly different behaviour

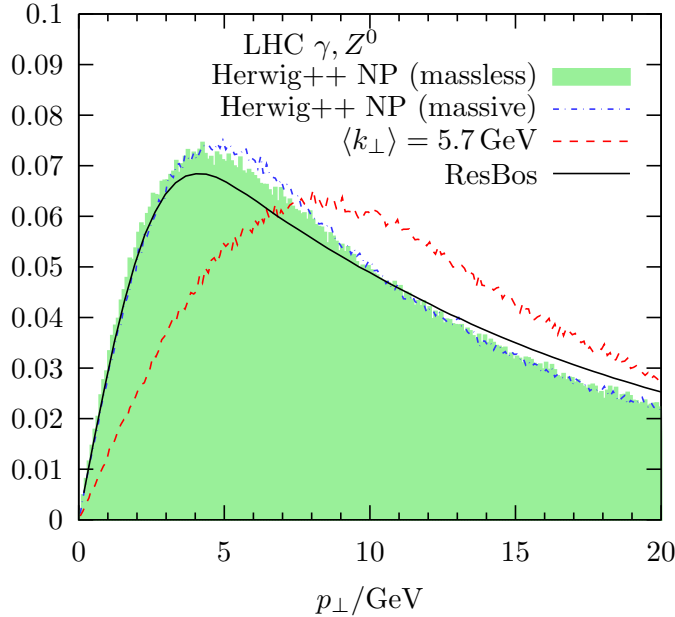


Figure 6: Vector boson p_{\perp} distribution at the LHC. Our model is compared to the extrapolation of Gaussian intrinsic k_{\perp} to LHC energies and the result from ResBos.

from our prediction. We predict a slightly more prominent peak and a stronger suppression towards larger transverse momenta. The same trend is already visible when comparing both approaches to Tevatron data although both are compatible with the data within the given error band. Both computations match the data well at large transverse momenta as they rely on the same hard matrix element contribution for single hard gluon emission. We want to stress the remarkable feature that we both predict the same peak position with these models. This is quite understandable as both models are built on the same footing: extra emission of soft gluons. A comparison of ResBos to data from experiments at various energies was made in [37].

Furthermore (dashed, red) we see the Herwig++ result from only using intrinsic $\langle k_{\perp} \rangle = 5.7 \text{ GeV}$ as recommended in [22,23]. This large value stems from an extrapolation from lower energy data with the assumption that the average k_{\perp} will depend linearly on $\ln(M/\sqrt{S})$. The peak is seen to lie at a considerably higher value of transverse momentum. It would clearly be of interest to have experimental data to distinguish these two models of non-perturbative transverse momentum.

5.2 Non-perturbative final state radiation

As briefly discussed in the introduction, we want to stress that the approach of adding non-perturbative soft gluon radiation to the parton shower should be connected to the non-perturbative input that the parton shower is linked to in the initial state. We think of this radiation as originating from long-range correlations within the

coloured initial state.

We have checked the effect of the same model for final state radiation. We find a dramatic increase in the amount of soft radiation when we compare event shapes, simulated with our new model for soft emissions, to LEP data, which are described well by the default parton shower model. Using the default hadronization model, we observe a dramatic softening of the event shapes, leading to a poor description of data. However, the default hadronization model produces a considerable amount of transverse momentum smearing during cluster splitting and decay, and is tuned to data together with a parton shower model that does not have non-perturbative smearing. Therefore to turn on this smearing, without modifying, or at least retuning, the hadronization model, must lead to a significant amount of double-counting. It is an interesting question, which we reserve for future work, whether a good fit can be obtained with our model.

6. Conclusion

Aiming for a universal model of non-perturbative soft gluon radiation we have achieved a reasonable description of data at three different energies. We consider the model based on soft gluon radiation, much like the resummation program ResBos, to have a more meaningful physics input than simply extrapolating the Gaussian smearing of a primordial transverse momentum. Of course, if this model is universal, it should make predictions for other processes, such as jet and photon production. We plan to study these processes in more detail in the future.

We also found that using our model as the only non-perturbative ingredient in the simulation, i.e. removing the non-perturbative constituent parton masses that usually cut off the parton shower in Herwig++, gave a somewhat better description of the data. This lays open the speculation that perhaps, in some way, the two approaches could be combined, using our model for initial-state radiation, and the usual model, tuned to describe the final states of e^+e^- annihilation, for final-state radiation. We leave consideration of this combination to future work however.

Acknowledgments

We would like to thank the referee of the first version of our paper for stimulating a more detailed study of our results on the parton level. We are grateful to Fred Olness and our Herwig++ collaborators for fruitful discussions and to Stefan Berge and Pavel Nadolsky for help with the ResBos comparison. We gratefully acknowledge support from the EU Marie Curie ESRT Fellowship EUROTHEPHY, under contract number (MEST-CT-2005-020238) and EU Marie Curie Research Training Network MCnet (MRTN-CT-2006-035606) which made this project at CERN possible. One

of us (AS) also would like to acknowledge the EU Marie Curie Research Training Network HEPTOOLS (MRTN-CT-2006-035505) for additional support.

A. Herwig++ parameter settings

The study has been done with Herwig++ 2.2.0 [22, 23]. We ran with the default matrix element for γ, Z production with only initial state parton showers. We left final state parton showers and hadronic decays switched off as they were irrelevant for this study. The following parameters in release 2.2.0 are important to switch off the final state parton shower and to adjust the intrinsic p_\perp :

```
cd /Herwig/Shower
set SplittingGenerator:FSR No
set Evolver:IntrinsicPtGaussian 0.4*GeV
```

Our preferred result, as shown in Fig. 5, was obtained by setting

```
set AlphaQCD:NPAlphaS 5
set AlphaQCD:Qmin 3.0*GeV
set AlphaQCD:AlphaMaxNP 3
```

Here, “AlphaQCD:NPAlphaS 5” selects the quadratic non-perturbative model. The flat model would correspond to setting this parameter to 6. AlphaQCD:Qmin sets the value of $p_{\perp 0}$ and AlphaQCD:AlphaMaxNP directly sets the value $\alpha_S(0)$. As obtaining results for the parton level with very small masses and cutoffs was very computing intensive, we also modified the code in order to leave out the timelike showers from partons that have been radiated in the initial state shower.

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